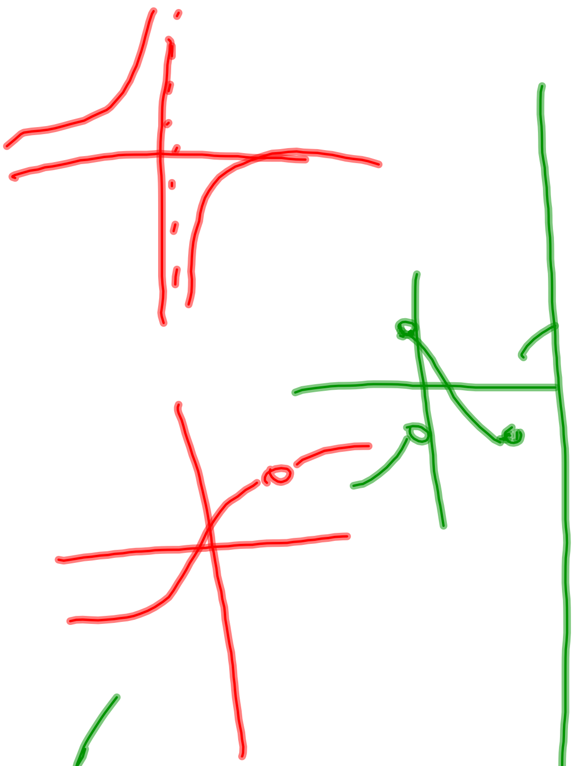
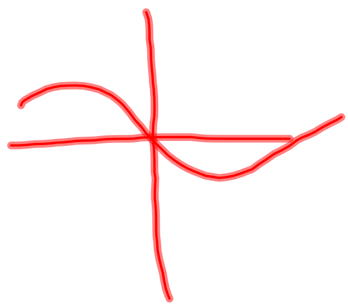
asymptote



noncontinuous



continuous

A $f^{\underline{n}}$ $f(x)$ is continuous at point $x=a$
iff "if and ONLY if"

i) $f(x)$ is defined at $x=a$

ii) $\lim_{x \to a} f(x)$ exists

iii) $f(a) = \lim_{x \to a} f(x)$

A fn $f(x)$ is continuous on an interval iff it is continuous at every pt in that interval.

7) f, g cont f^n $\overset{\text{"such that"}}{\ni}$ $f(2)=1$ and

$$\lim_{x\to 2}\left[f(x)+4g(x)\right]=13$$

Find a) $g(2) = \lim_{x\to 2} g(x)$

$= 3$

b) $\lim_{x\to 2} g(x)$

$$\lim_{x\to 2}\left[f(x)+4g(x)\right] = \lim_{x\to 2} f(x) + 4\lim_{x\to 2} g(x)$$

b/c f is cont

$$= f(2) + 4\lim_{x\to 2} g(x) = 13$$

$$\Rightarrow 1+4\lim_{x\to 2} g(x) = 13$$

$$\Rightarrow 4\lim_{x\to 2} g(x) = 12$$

$$\Rightarrow \lim_{x\to 2} g(x) = 3$$

f is continuous at $x = c$

iff "if and only if"

i) f is defined at c

ii) $\lim_{x \to c} f(x)$ exists

iii) $\lim_{x \to c} f(x) = f(c)$

10) Find formulas for some f^n that are cont on $(-\infty, 0), (0, \infty)$ but NOT $(-\infty, \infty)$

$y = \frac{1}{x}$

$y = \frac{1}{x^2}$

$y = \begin{cases} 3, & x \neq 0 \\ 7, & x = 0 \end{cases}$

$y = \frac{1}{x^3}$

$y \cancel{=} \{3, x \neq 0$

$y = \frac{1}{x^4}$

$\vdots$

14) $f(x) = (x-5)^{17}$ All pollynomials are cont. everywhere

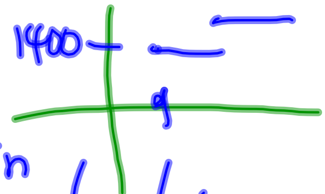
find xs for which $f(x)$ is not cont.

VERY much like those PC problems that ask for domain

16) $f(x) = \frac{x}{x^2-1}$ find all x's where f(x) is NOT cont.

All rational $f^{n's}$ are cont everywhere in their domain. $x=-1$, $x=1$

$x^2-1=0$ $x^2=1$ $x=\pm 1$

12) cont?  

not cont.

a) Earth's pop as a f^n of time

usually cont.

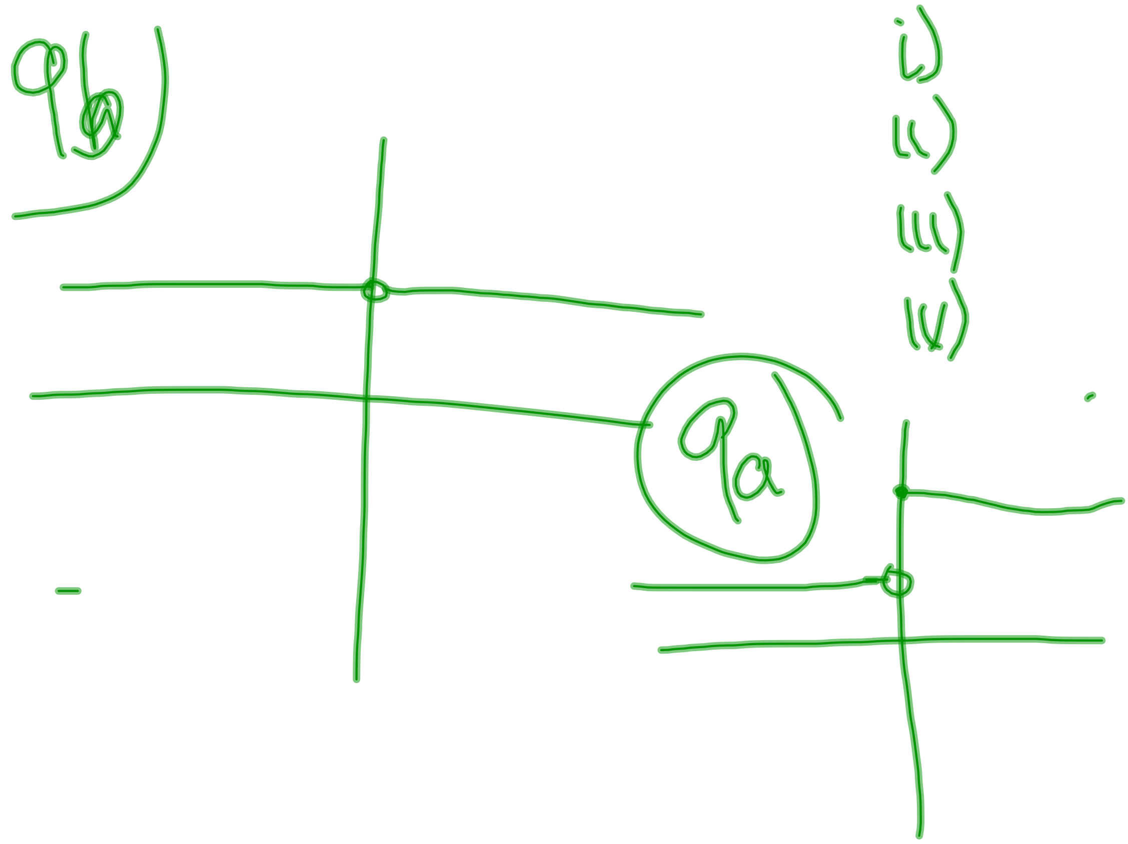
b) exact heisht as f^n of time

not cont

c) cost of taxi ride as f^n of distance travelled

d) volume of melting ice cube as fn of time.

cont or not cont depends on depth of your knowledge & matter



"continuous from the right"
at $x=c$

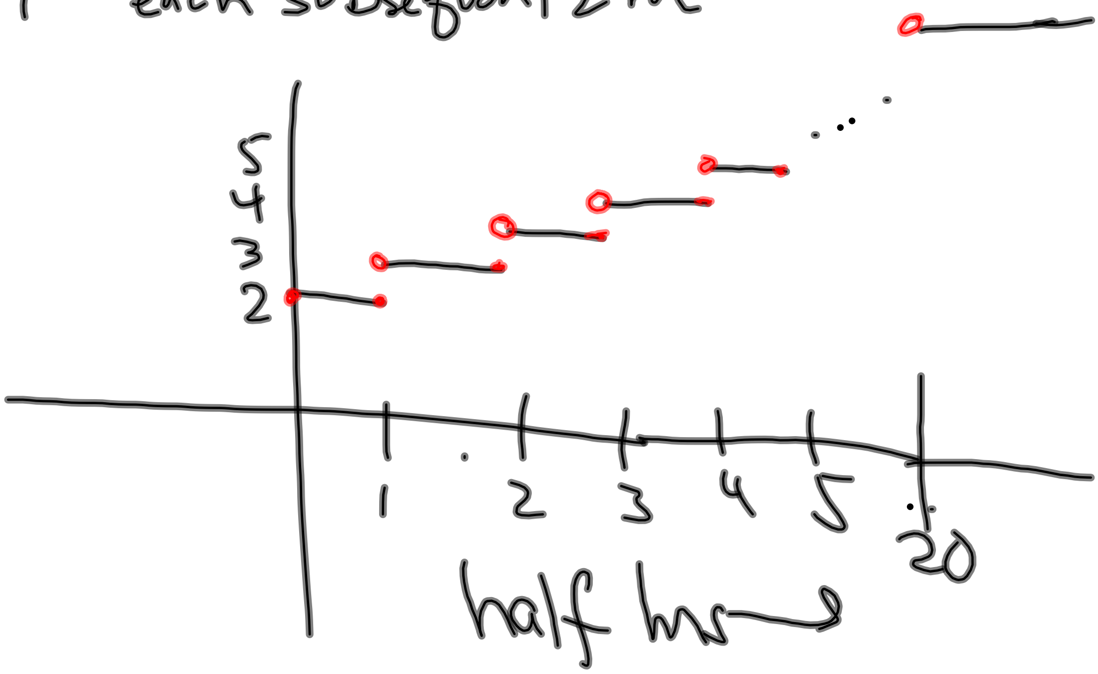
i) $f(x)$ is def^n at $x=c$

ii) $\lim_{x\to c^+} f(x)$ exists

iii) $\lim_{x\to c^+} f(x) = f(c)$

11) \$2 first $\frac{1}{2}$ hr. \$10 max

\$1 each subsequent $\frac{1}{2}$ hr



25) Find a value for k (constant) that will make $f(x)$ cont. everywhere

a) $f(x) = \begin{cases} 7x-2, & x \leq 1 \\ kx^2, & x > 1 \end{cases}$

I) this f^n can only be discontinuous at $x=1$

$y=0$ $\quad c_n x^n + c_{n-1} x^{n-1} + \cdots + c_1 x + c_0$

II $f(x)$ is cont at $x=1$ iff

i) $f(1)$ exists ✓

ii) $\lim_{x \to 1} f(x)$ exists

$\lim_{x \to 1^+} f(x)$ $\qquad$ $\lim_{x \to 1^-} f(x) =$

$= \lim_{x \to 1^+} kx^2 = k$ $\qquad$ $\lim_{x \to 1^-} 7x-2 = 7(1)-2 = 5$

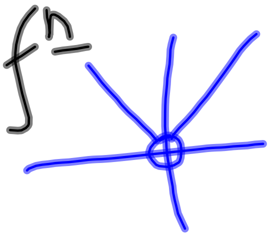
therefore ($\therefore$) $\lim_{x \to 1} f(x)$ exists when $k=5$

iii) $\lim_{x \to 1} f(x) = 5 = f(1)$

$\therefore$ $f(x)$ is cont when $k=5$

10) find formulas for some $f^{\underline{n}}$ that are continuous on

$(-\infty,0)$, $(0,\infty)$ but not on $(-\infty,\infty)$



$f(x)=\begin{cases} x, & x>0 \\ -x, & x<0 \end{cases}$

$f(x)=\frac{1}{x}$

2)

[1,2)

[1,2]
no (because of x=2)

(1,2)
yes

A) [1,3]
HOLE at x=2

B) (1,3)
NO

7) Spose that f, g cont "such that" $\ni f(2)=1$

and $\lim_{x \to 2} [f(x)+4g(x)] = 13$

a) $g(2) =$ [since g is cont AND $\lim_{x \to 2} g(x) = 3$, then $g(2)=3$]

b) $\lim_{x \to 2} g(x) =$

$\lim_{x \to 2} [f(x)+4g(x)] =$

$\lim_{x \to 2} f(x) + 4 \lim_{x \to 2} g(x)$

b/c $f(2)=1$ AND f cont.

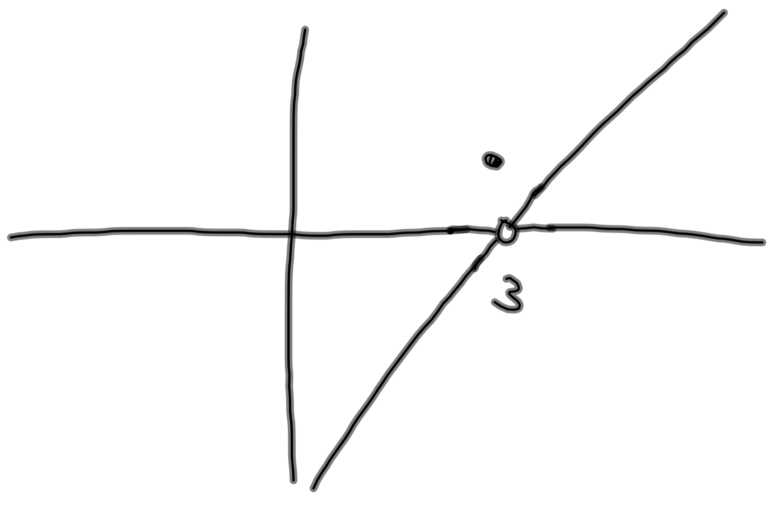
$= 1 + 4 \lim_{x \to 2} g(x) = 13$

$\therefore$ "therefore" $\lim_{x \to 2} g(x) = \frac{13-1}{4} = 3$

9) c) f is NOT cont at x=3

but if f(3)=0 instead of 1

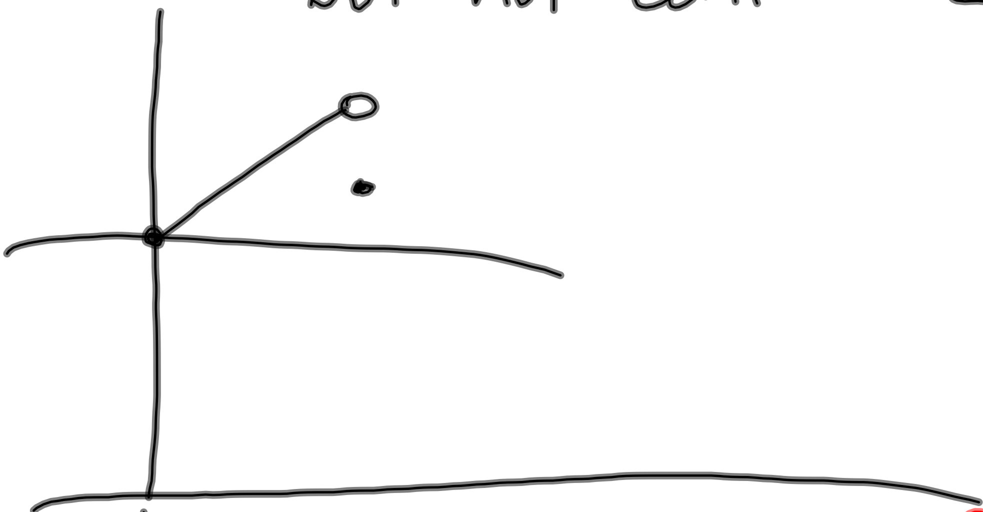
then f is cont at 3



9d) f cont on $[0,3)$

and $\text{def}^{\underline{n}}$ on $[0,3]$

but not cont on $[0,3]$



A fn $f(x)$ is continuous from the right / from the left

at $x = a$ iff

i) $f(a)$ exists [[$f(x)$ is defined at $x=a$]]

ii) $\lim\limits_{x \to a^{\pm}} f(x)$ exists

iii) $\lim\limits_{x \to a^{\pm}} f(x) = f(a)$

12) a) Earth's population as a fn of time.

not cont

F

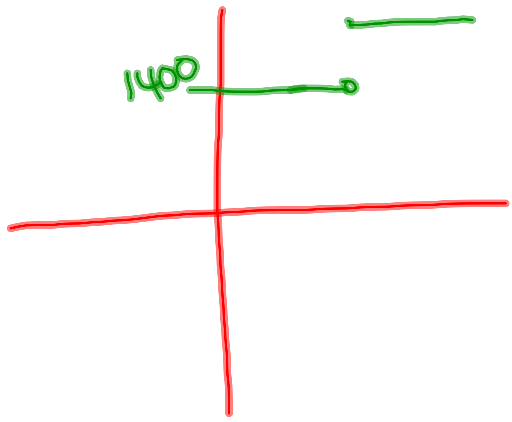
b) Your exact height as a function of time.

usual continuous

c) Cost of a taxi ride as a fn of distance

not cont

d) volume of melting ice cube as fn of time



cont or not depends on your depth of knowledge of matter.

25a) $f(x)=\begin{cases} 7x-2 & x\le 1 \\ kx^2 & x>1 \end{cases}$

Find k (constant) that makes f(x) cont. everywhere.

I) f(x) will be continuous everywhere except possibly x=1

II) f(x) is cont. at x=1 iff

i) f(x) is defn at x=1

yes

ii) $\lim_{x\to 1} f(x)$ exists?

$\lim_{x\to 1^+} f(x)=$

$\lim_{x\to 1^+} kx^2 = k$

$\lim_{x\to 1^-} f(x)=$

$\lim_{x\to 1^-} 7x-2 = 7(1)-2 = 5$

$\therefore$ (therefore) $\lim_{x\to 1} f(x)$ exists IF

$k=\lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} f(x) = 5$

so $k=5$

iii) if $k=5$

then $\lim_{x\to 1} f(x) = 5 = f(1)$

$\therefore$ f(x) is cont. when $k=5$